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Super λ_3-optimality of regular graphs

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ABSTRACT

Let $G = (V, E)$ be a connected graph. An edge set $S \subset E$ is a 3-restricted edge cut, if $G - S$ is disconnected and every component of $G - S$ has at least three vertices. The 3-restricted edge connectivity $\lambda_3(G)$ of G is the cardinality of a minimum 3-restricted edge cut of G. A graph G is λ_3-connected, if 3-restricted edge cuts exist. A graph G is called λ_3-optimal, if $\lambda_3(G) = \xi_3(G)$, where $\xi_3(G) = min\{|[X, \overline{X}]| : X \subseteq V, |X| = 3, G[X] \text{ is connected}\}$, $[X, \overline{X}]$ is the set of edges of G with one end in X and the other in $\overline{X}$ and $\overline{X} = V - X$. Furthermore, if every minimum 3-restricted edge cut is a set of edges incident to a connected subgraph induced by three vertices, then G is said to be super 3-restricted edge connected or super-λ_3 for simplicity. In this paper we show that let G be a k-regular connected graph of order $n \geq 6$, if $k \geq \lfloor n/2 \rfloor + 3$, then G is super-λ_3.

1. Introduction

It is well known that graph theory plays a key role in the analysis and design of reliable or invulnerable networks. A network is often modeled by a graph $G = (V, E)$ with the vertices representing nodes such as processors or stations, and the edges representing links between the nodes. One fundamental consideration in the design of networks is reliability [1–5]. An edge cut of a connected graph G is a set of edges whose removal disconnects G. The *edge connectivity* $\lambda(G)$ of G is the minimum cardinality of an edge cut S of G. The edge connectivity $\lambda(G)$ is an important feature determining reliability and fault-tolerance of the network [6–8]. The following model was proposed in [9]. Let $G = (V, E)$ be a graph with the vertices reliable, but the edges may fail independently with the same probability $\rho \in (0, 1)$. One measure of the network reliability is the probability $P(G)$ of G being disconnected:

$$P(G) = \sum_{i=\lambda}^{\epsilon} m_i(G)\rho^i(1-\rho)^{\epsilon-i},$$

where ϵ is the number of edges in G, $m_i(G)$ is the number of edge cuts of size i, λ is the edge connectivity. $P(G)$ is a polynomial on variable ρ, and is called unreliability polynomial. It can be seen that the smaller $P(G)$ the more reliable of the network. In general, to determine $P(G)$ is difficult [9]. When ρ is sufficiently small, the minimum of $P(G)$ can be obtained by maximizing λ first and then minimizing $m_\lambda(G), m_{\lambda+1}(G), \ldots, m_\epsilon(G)$ sequentially [10]. In the model, the parameter λ, however, has an obvious deficiency, that is, they tacitly assume that all edges incident with the same vertex of G can potentially fail at the same time, which happens almost impossible in the practical applications of networks. In other words, in the definitions of $\lambda(G)$, absolutely no restrictions are imposed on the components of $G - S$. Consequently, the measurement is inaccurate

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for large-scale parallel processing systems in which all processors adjacent to or all links incident with the same processor cannot fail at the same time. To compensate for this shortcoming, it would seem natural to generalize the notion of the classical connectivity by imposing some conditions or restrictions on the components of $G - S$.

Following this idea, k-restricted edge connectivity were proposed [11]. An edge set $S \subset E$ is said to be a *k-restricted edge cut*, if $G - S$ is disconnected and every component of $G - S$ has at least k vertices. The *k-restricted edge connectivity* of G, denoted by $\lambda_k(G)$, is the cardinality of a minimum k-restricted edge cut of G. If S is a k-restricted edge cut and $|S| = \lambda_k(G)$, then we call S a *λ_k-cut*. Not all graphs have k-restricted edge cuts. A connected graph G is called *λ_k-connected*, if it has a k-restricted edge cut. If S is a *λ_k-cut*, then $G - S$ has only two connected components. It is easy to see that if G is λ_k-connected ($k \geq 2$), then it is also λ_{k-1}-connected and $\lambda_{k-1}(G) \leq \lambda_k(G)$. It seems that the larger $\lambda_k(G)$ is, the more reliable the network is [12,13]. So, we expect $\lambda_k(G)$ to be as large as possible. Let

$$\xi_k(G) = min\{\omega(X) : X \subseteq V, |X| = k, G[X] \text{ is connected}\},$$

where $[X, \overline{X}]$ is the set of edges of G with one end in X and the other in $\overline{X}$, $\overline{X} = V - X$ and $\omega(X) = |[X, \overline{X}]|$. It has been shown that $\lambda_k(G) \leq \xi_k(G)$ holds for many graphs [14,15]. Then G is said to be *λ_k-optimal*, if $\lambda_k(G) = \xi_k(G)$. Furthermore, G is called *superk-restricted edge connected* or *super-λ_k*, if every λ_k-cut of G isolates one connected subgraph of order k, that is, every λ_k-cut is a set of edges adjacent to a certain connected subgraph of order k. Clearly, $\lambda_1 = \lambda$, $\lambda_2 = \lambda'$, $\xi_1 = \delta$ and $\xi_2 = \xi$ is the minimum edge degree. If G is super-λ_k, then it is λ_k-optimal. However, the converse is not true. The cycle of length $n \geq 2k + 2$ is a counterexample.

Esfahanian and Hakimi proved the existence of restricted edge cuts and upper bound for the restricted edge connectivity.

Theorem 1.1 (*Esfahanian and Hakimi [16]*). *For any connected graph G with at least four vertices which is not isomorphic to the star $K_{1,n-1}$, $\lambda'(G)$ is well defined. Furthermore, $\lambda'(G) \leq \xi(G)$, where $\xi(G) = min\{\xi(e) = d(u) + d(v) - 2 : e = uv \in E\}$ is the minimum edge degree of G.*

For $\lambda_3(G)$, It has been shown by Meng et al. that

Theorem 1.2 (*Meng and Ji [12]*). *If G is a λ_3-connected graph, then $\lambda_3(G) \leq \xi_3(G)$.*

For graph-theoretical terminology and notation not defined here we follow [17]. All graphs considered in this paper are simple, finite and undirected.

Let $G = (V, E)$ be a connected graph, $d_G(v)$ be the degree of a vertex v in G (simply $d(v)$), and $\delta(G)$ be the minimum degree of G. Moreover, for $S \subset V$, $G[S]$ is the subgraph induced by S. We use $G - S$ to denote the subgraph of G induced by the vertex set of $V \setminus S$ and $\overline{S} = V - S$. If $u, v \in V$, $d(u, v)$ denotes the length of a shortest (u, v)-path. And the *diameter* is $dm(G) = max\{d(u, v) : u, v \in V\}$. The *girth* g of G is the minimum length of cycles in G and P_n is the path of n vertices. Define the inverse degree of a graph G with no isolated vertices as

$$R(G) = \sum_{v \in V} \frac{1}{d(v)}.$$

The inverse degree first attracted attention through conjectures of the computer program Graffiti [18].

Different authors proposed sufficient conditions for a graph to be λ_k-optimal and super-λ_k.

Theorem 1.3. *Let G be a connected graph with order n,*

(1) *If $n \leq 2\delta + 1$, then $\lambda = \delta$* [19].
(2) *If G is a graph with $\delta(G) \geq (n + 1)/2$, then G is super-λ* [20].
(3) *Let k be a positive integer, and G a connected graph on $n \geq 2k$ vertices. Suppose that $d(u) + d(v) \geq n + 2k - 3$, for every pair of nonadjacent vertices u and v in G. Then G is λ_k-optimal* [15].
(4) *If*

$$R(G) < 2 + \frac{n - 2\delta}{(n - \delta)(n - \delta - 1)},$$

then G is super-λ for $\delta \geq 2$ and $n \geq 2\delta + 2$ [21].
(5) *Let G be a λ'-connected graph, minimum degree $\delta \geq 3$ and girth g. If $dm(G) \leq g - 3$, then G is super-λ'* [22].
(6) *Let G be a λ_3-connected triangle-free graph. If $|N(u) \cap N(v)| \geq 3$ for all pairs u, v of nonadjacent vertices, then G is λ_3-optimal* [23].
(7) *Girth $g \geq 4$ and $\delta \geq 3$. If $dm(G) \leq g - 4$, then G is super-λ_3* [24].

In this paper, we show the following theorem.

Theorem 1.4. *Let G be a k-regular connected graph of order $n \geq 6$. If $k \geq \lfloor n/2 \rfloor + 3$, then G is super-λ_3.*

2. Properties of fragments

Let $S = [X, \overline{X}]$ be a λ_3-cut, we call X a *fragment* of G. That is a fragment is a subset $X \subset V$ such that $\overline{X} \neq \emptyset$ and $\omega(X) = \lambda_3$. Note that the fragment is connected and X is a fragment if and only if $\overline{X}$ is also a fragment. A *normal fragment* is a fragment with order at most $\lfloor n/2 \rfloor$. An *atom* is the minimum fragment, whose cardinality is denoted by $\eta(G)$. And we know that $\eta(G) \leq \lfloor n/2 \rfloor$.

If G is a 2-regular connected graph and super-λ_3, then it is C_6 or C_7. Hence we assume $k \geq 3$ for k-regular connected graphs in the following discussion.

Lemma 2.1. *Let X and Y be two different fragments of G. If*

(a) $|X \cap Y| \geq 3$, *and*
(b) $\omega(X \cap Y) \leq \lambda_3$, *then $X \cap Y$ is a fragment.*

Proof. Let $A = X \cap Y$, $B = X \cap \overline{Y}$, $C = \overline{X} \cap Y$ and $D = \overline{X} \cap \overline{Y}$. It suffices to show that both A and $\overline{A}$ are connected.

1. We will prove that $\overline{A} = \overline{X} \cup \overline{Y}$ is connected.
   Since $\overline{X}$ and $\overline{Y}$ are both connected fragments. If $D \neq \emptyset$, then $\overline{A}$ is connected. Hence $D = \emptyset$. If $[B, C] = \emptyset$, then $|[X, \overline{Y}]| = |[B, A]| = |[A, C]| = \lambda_3$ and $\omega(A) = |[B, A]| + |[A, C]| = 2\lambda_3 > \lambda_3$, a contradiction. Hence we get $[B, C] \neq \emptyset$, and $\overline{A}$ is connected.
2. We will prove that $A = X \cap Y$ is connected.
   Let A_i be the connected components of A for $i = 1, \ldots, m$ (≥ 2). If every A_i is an isolated vertex, then $m \geq 3$ and $\omega(A) = \sum_{v \in A} d(v) \geq 3k > \xi_3 \geq \lambda_3$, a contradiction. Hence there is a $|A_j| \geq 2$. Since $|\overline{A}| \geq 3$ and by item 1 $\overline{A}$ is connected, if $|A_j| \geq 3$, then $\omega(A) = |[A_j, \overline{A}]| + |[A \setminus A_j, \overline{A}]| \geq \lambda_3 + 1 > \lambda_3$, a contradiction. Hence for each A_i we have $1 \leq |A_i| \leq 2$. We can also obtain a contradiction. □

Lemma 2.2. *Let X and Y be two different normal fragments of G. If $|X \cap Y| \geq 3$, then $X \cap Y$ and $\overline{X} \cap \overline{Y}$ are fragments.*

Proof. Let A, B, C and D be the same in the proof of Lemma 2.1. By Lemma 2.1, it suffices to show that $|D| \geq |A| \geq 3$ and $\omega(A), \omega(D) \leq \lambda_3$. □

Claim 1. $|D| \geq |A| \geq 3$.

Since $|X| \leq \lfloor n/2 \rfloor \leq |\overline{X}|$ and $|Y| \leq \lfloor n/2 \rfloor \leq |\overline{Y}|$, we have $|A| + |C| = |Y| \leq \lfloor n/2 \rfloor \leq |\overline{X}| = |D| + |C|$. It follows that $|D| \geq |A| \geq 3$.

Claim 2. $\omega(A) \leq \lambda_3$.

First, we will prove that $|[A, B]| \leq |[D, B]|$. If $|[A, B]| > |[D, B]|$, then $\omega(D) = |[D, Y]| + |[D, B]| < |[D, Y]| + |[A, B]| + |[B, C]| = |[\overline{Y}, Y]| = \lambda_3$. By Claim 1 and Lemma 2.1 D is a fragment. That is $[D, \overline{D}]$ is a 3-restricted edge cut with $\omega(D) = \lambda_3$, contradicting to the inequality. Then we have $\omega(A) = |[A, \overline{X}]| + |[A, B]| \leq |[A, \overline{X}]| + |[D, B]| + |[B, C]| = |[X, \overline{X}]| = \lambda_3$.

Claim 3. $\omega(D) \leq \lambda_3$.

By Lemma 2.1 and Claims 1 and 2, A is a fragment of $\omega(A) = \lambda_3$. Hence $\omega(A) + \omega(D) = \omega(X) + \omega(Y) - 2|[B, C]| \leq 2\lambda_3$ and $\omega(D) \leq \lambda_3$.

Corollary 2.3. *Let X and Y be two different atoms of G, then $|X \cap Y| \leq 2$.*

Proof. If $|X \cap Y| \geq 3$, then by Lemma 2.2 $X \cap Y$ is a fragment, but $X \cap Y \neq X$ and $|X| = |Y|$. Hence X contains a proper subgraph as a fragment, a contradiction to the definition of atom.

Lemma 2.4 (*Mader [25]*)**.** *If G is a connected graph which is vertex transitive and K_4-free, then $\delta(G) = \kappa(G)$.*

Lemma 2.5. *Let G be a connected graph which is vertex transitive and K_4-free, X and Y be two different atoms of G. If $X \cap Y \neq \emptyset$, then $X, Y \cong K_3$ or P_3.*

Proof. If $X, Y \ncong K_3$ and P_3, then $|X| = |Y| \geq 4$. By Corollary 2.3 $|X \cap Y| \leq 2$.

If $X \cap Y = \{u\}$, then $|X \cap \overline{Y}| = |\overline{X} \cap Y| = |X| - |X \cap Y| \geq 3$. First, if $|[u, \overline{X} \cap Y]| \geq |[u, X \cap \overline{Y}]|$, then

$$\begin{aligned}
\omega(X \cap \overline{Y}) &= |[u, X \cap \overline{Y}]| + |[X \cap \overline{Y}, \overline{X} \cap Y]| + |[X \cap \overline{Y}, \overline{X} \cap \overline{Y}]| \\
&\leq |[u, \overline{X} \cap Y]| + |[X \cap \overline{Y}, \overline{X} \cap Y]| + |[X \cap \overline{Y}, \overline{X} \cap \overline{Y}]| + |[u, \overline{X} \cap \overline{Y}]| \\
&\leq \omega(X) = \lambda_3.
\end{aligned}$$

By Lemma 2.1, $X \cap \overline{Y}$ is a fragment. But it is also a proper subgraph of atom X, a contradiction. Similarly, if $|[u, \overline{X} \cap Y]| \leq |[u, X \cap \overline{Y}]|$, then $\overline{X} \cap Y$ is a fragment, again a contradiction.

We let $X \cap Y = \{u, v\}$. If $G[X \cap Y]$ is not connected, then $X - u$ or $X - v$ is connected. Say, $X - v$ is connected. First, we assume that $Y - v$ is connected. If $|[v, C]| \geq |[v, B]|$, then

$$\begin{aligned}\omega(B+u) &= |[B, C]| + |[u, C]| + |[B, v]| + |[B, D]| + |[u, D]| \\ &\leq |[B, C]| + |[u, C]| + |[v, C]| + |[B, D]| + |[u, D]| + |[v, D]| \\ &= \omega(X) = \lambda_3.\end{aligned}$$

By Lemma 2.1 $B + u$ is a fragment contained in atom X, a contradiction. If $|[v, C]| \leq |[v, B]|$, then $C + u$ is a fragment, again a contradiction. If $Y - u$ is connected, then it is similar to the above case, we can get a contradiction.

Hence let $G[X \cap Y]$ be connected. It is an easy exercise to prove that the subgraph induced by an atom is also connected and vertex transitive (we can see [26]). And since $|X| \geq 4$, according to Lemma 2.4, we have $\kappa(X) = \delta(X) \geq 2$. That is, $X - v$ is connected. Then it is similarly to the above case, we can also get a contradiction. □

Corollary 2.6. *Let G be a connected k-regular graph with order $n \geq 6$ which is vertex transitive and K_4-free:*

(1) *every atom is isomorphic to K_3 or P_3, or*
(2) *no atom is K_3 and P_3, and the intersection of any two atoms is empty.*

Lemma 2.7. *Let G be a connected k (≥ 5)-regular graph which is vertex transitive and K_4-free. The following three statements are equivalent to each other.*

(1) *G is not λ_3-optimal.*
(2) *$\eta(G) \geq k - 1$.*
(3) *Atoms of G are pairwise disjoint.*

Proof. (1) ⇒ (2). Let X be an atom. Since G is not λ_3-optimal. If $\xi_3(G) = 3k - 6$, then we have

$$\begin{aligned}&3k - 6 > \lambda_3 = \omega(X) \geq k|X| - |X|(|X| - 1) \\ &(|X| - 3)(|X| - k + 2) > 0.\end{aligned}$$

By Corollary 2.6, $|X| > 3$. Hence $\eta(G) \geq k - 1$.

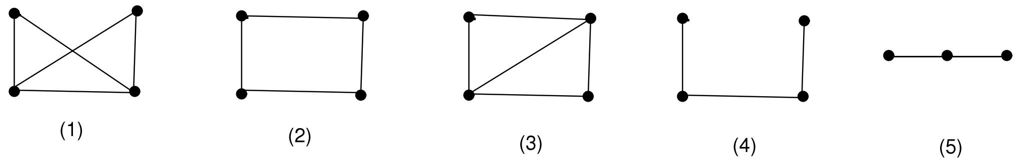
If $\xi_3(G) = 3k - 4$, then we have

$$\begin{aligned}&3k - 4 > \lambda_3 = \omega(X) \geq k|X| - |X|(|X| - 1) + 2 \\ &(|X| - 3)(|X| - k + 2) > 0.\end{aligned}$$

By Corollary 2.6, $|X| > 3$. Hence $\eta(G) \geq k - 1$.

(2) ⇒ (3). Let X, Y be two distinct atoms. Because of $|X| = \eta(G) \geq k - 1 \geq 4$, that is $X \ncong K_3, P_3$. According to Corollary 2.6 $X \cap Y = \varnothing$.

(3) ⇒ (1). Since G is K_4-free. G contains at least one of the following graphs as an induced subgraph.



If G contains (1), then we can find two atoms which are not disjoint. It is similar that G contains (2), (3) or (4). If G contains (5), then let xyz be an induced path. Since $d(y) \geq 5$, if there are two vertex $y_1, y_2 \in N(y)$ such that $xy_1, xy_2 \in E$, then G contains (3). Hence there are two induced P_3's which are not disjoint. □

3. Super λ_3-optimality

Excluding all atoms, the smallest fragments with cardinality at most $\lfloor n/2 \rfloor$ in G are called *superatoms*. The following lemma is obvious.

Lemma 3.1. *Let G be a connected k-regular λ_3-optimal graph. Then G is super-λ_3 if and only if it has no superatoms.*

Lemma 3.2. *Let G be a connected k-regular λ_3-optimal graph of girth g. If X is a superatom, then $|X| \geq k - 2$. Furthermore, X is K_{k-2} if girth $g = 3$.*

Proof. Suppose that G is λ_3-optimal. If $g = 3$, then we have

$$\begin{aligned}&3k - 6 = \xi_3 = \lambda_3 = \omega(X) = k|X| - \sum_{x \in X} d(x) \geq k|X| - |X|(|X| - 1) \\ &(|X| - k + 2)(|X| - 3) \geq 0.\end{aligned}$$

Since $|X| \geq 4$, we get $|X| \geq k - 2$. $|X| - k + 2 = 0$ if and only if $\sum_{x \in X} d(x) = |X|(|X| - 1)$. That is, $X \cong K_{k-2}$.

If $g \geq 4$, then

$$3k - 4 = \xi_3 = \lambda_3 = \omega(X) = k|X| - \sum_{x \in X} d(x) \geq k|X| - |X|(|X| - 1) + 2$$

$$(|X| - k + 2)(|X| - 3) \geq 0.$$

Since $|X| \geq 4$, we have $|X| \geq k - 2$. □

By Theorem 1.3(3), we can obtain the following corollary.

Corollary 3.3. *Let G be a connected k-regular graph of order $n \geq 6$. If $k \geq \lfloor n/2 \rfloor + 2$, then G is λ_3-optimal.*

Theorem 3.4. *Let G be a k-regular connected graph of order $n \geq 6$. If $k \geq \lfloor n/2 \rfloor + 3$, then G is super-$\lambda_3$.*

Proof. According to Corollary 3.3 G is λ_3-optimal. If G is not super-λ_3, then let X be a superatom and by Lemma 3.2, we have

$$\lfloor n/2 \rfloor \geq |X| \geq k - 2 \geq \lfloor n/2 \rfloor + 3 - 2 \geq \lfloor n/2 \rfloor + 1,$$

a contradiction. □

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